Improving Poker Winnings with Game Theory

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Abstract

This paper delves into the limitations of relying solely on traditional pot odds as a strategy in poker and advocates for the adoption of a more comprehensive game theory approach. While pot odds have long been a staple in poker decision-making, they often fall short in capturing the complexity of the game dynamics and fail to provide optimal strategies in certain scenarios. The paper highlights the need for players to embrace game theory, offering a more nuanced understanding of poker strategies that considers not only immediate odds but also opponent behaviors, ranges, and potential future actions. The analysis presents a concrete example of the mixed strategy Nash equilibrium, showcasing how a player can make optimal decisions by probabilistically selecting different actions in a given situation. By employing mixed strategies, players introduce an element of unpredictability into their gameplay, making it challenging for opponents to exploit specific patterns. The paper emphasizes the importance of understanding and implementing mixed strategies to achieve equilibrium and gain a strategic edge in poker. To support these theoretical concepts, the paper introduces examples of software tools that aid players in analyzing mixed strategy Nash equilibria.

1 Introduction

To those new to the game, poker begins with each player being dealt two cards (hole cards) out of the standard 52-card deck. Players keep their hand hidden from the other players. In Texas Hold'em, the game starts with two players posting forced bets called the "small blind" and the "big blind", initiating the betting pot. This first round of betting is called pre-flop. Starting with the players next to the big blind, each player has three available actions: call (match the previous bet), raise (bet an additional amount on top of the opponent's bet), or fold (discard their hand and forfeit the round). When all bets and raises have been called, players also have the option to check (pass the action to the next player without putting additional money into the pot). When all players check, the pre-flop round ends and three community cards (called the "flop") are dealt face up in the center of the table, and they are seen by all players. A new round of betting starts with the small blind, where each player can check, bet, call, raise, or fold. After this round ends, a fourth community card, called the

	A	K	Q	J	T	9	8	7	6	5	4	3	2
A	85%	68%	67%	66%	66%	64%	63%	63%	62%	62%	61%	60%	59%
K	66%	83%	64%	64%	63%	61%	60%	59%	58%	58%	57%	56%	55%
Q	65%	62%	80%	61%	61%	59%	58%	56%	55%	55%	54%	53%	52%
J	65%	62%	59%	78%	59%	57%	56%	54%	53%	52%	51%	50%	50%
T	64%	61%	59%	57%	75%	56%	54%	53%	51%	49%	49%	48%	47%
9	62%	59%	57%	55%	53%	72%	53%	51%	50%	48%	46%	46%	45%
8	61%	58%	55%	53%	52%	50%	69%	50%	49%	47%	45%	43%	43%
7	60%	57%	54%	52%	50%	48%	47%	67%	48%	46%	45%	43%	41%
6	59%	56%	53%	50%	48%	47%	46%	45%	64%	46%	44%	42%	40%
5	60%	55%	52%	49%	47%	45%	44%	43%	43%	61%	44%	43%	41%
4	59%	54%	51%	48%	46%	43%	42%	41%	41%	41%	58%	42%	40%
3	58%	54%	50%	48%	45%	43%	40%	39%	39%	39%	38%	55%	39%
2	57%	53%	49%	47%	44%	42%	40%	37%	37%	37%	36%	35%	51%

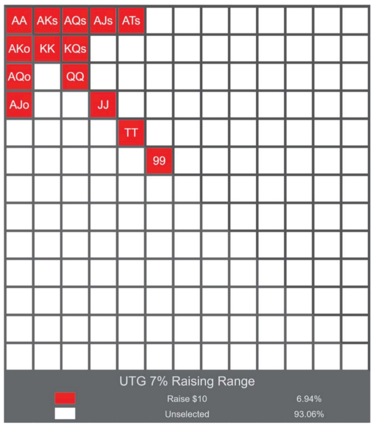
Figure 1: https://www.actionnetwork.com/casino/poker/top-20-hole-cards

"turn", is dealt face up, followed by another betting round. Lastly, a fifth and final community card, called the "river", is dealt face up, and the last betting round takes place. If more than one player remains after the final betting round, a showdown occurs, where players reveal their hole cards, and the best hand is determined based on the best five-card combination using any combination of the hole cards and the community cards. The player with the best hand wins the pot.

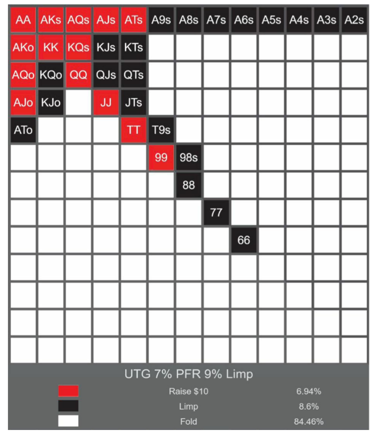
2 Poker Equity

Given the ranking of the best five-card combinations, it is clear that the initial hole cards have different strengths, even in the pre-flop round. This strength is called Poker Equity, the chance a player has to win the hand at the end. The starting hand's strength analysis in Figure 1 shows the odds of winning at showdown based on pre-flop cards.

To assess whether a calling a bet is worthwhile given one's hole cards, we employ expected value calculations: Denote:
p = probability of winning with one's hand
X = pot value
Y = call amount
Fold $EV = 0$
Call $EV = pX - (1-p)Y = p(X+Y) - Y$
If $p > Y/(X+Y)$, then it is statistically better to call the bet.



(a) Modern poker theory: building an unbeatable strategy based on GTO principles, page 27



(b) Modern poker theory: building an unbeatable strategy based on GTO principles, page 28

If $p < Y/(X+Y)$ it is statistically better to fold.
Lastly, if $p = Y/(X+Y)$, the player is indifferent between folding or calling.

Likewise, if a player places a bet, they reveal some information about the strength of their hand. In “Modern Poker Theory”, Michael Acevedo, a professional online tournament player, divides the initial pairs in ranges, according to their strength and to the degree of revealed information. Range 1 includes all possible combinations of two cards, and is used when no information is known about an opponent’s hand. Range 2 consists of the strongest 7% hands, with which almost every player raises from any position, and is represented by Figure (a).

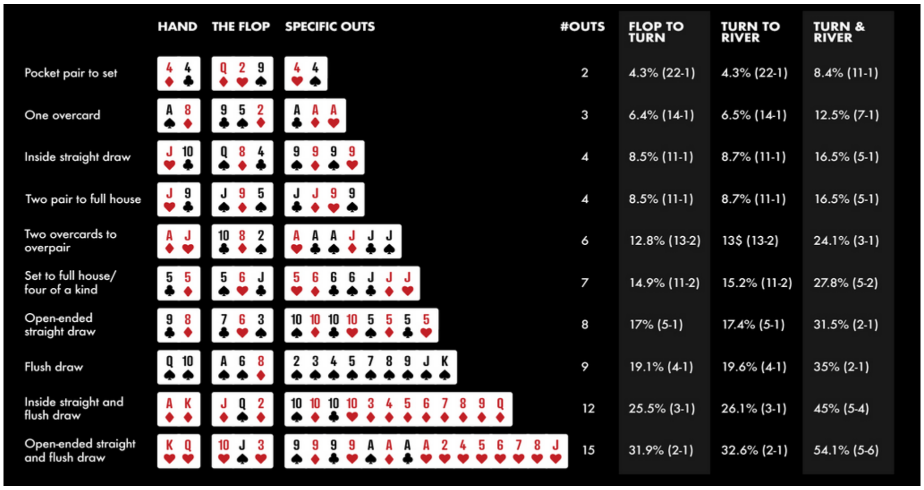
Range 3 includes medium strength hands that are not good enough to raise, but might be good enough to call to see the flop, represented by Figure (b).

It is important to note that hand ranges are subjective to different players’ strategies. Some players will play more aggressively, meaning that they will raise even medium strength hands, while others tend to limp weaker hands. Therefore it is useful to pick on opponents’ ranges early on.

3 Pot Odds

Instead of calculating the probability of winning throughout the game, a simpler approach is to calculate one’s Pot Odds, defined as the ratio of the current size of the pot to the cost of the call. In other words, it represents the reward-to-risk ratio of calling the bet. Since it is not useful when comparing equity percentages, the cost of the call is divided by the sum of the sizes of the pot and call. Pot Odds = Reward : Risk = Pot/ Pot + Call = Risk/(Reward + Risk)

Pot odds can be useful when a player has a drawing hand, since comparing



	HAND	THE FLOP	SPECIFIC OUTS	#OUTS	FLOP TO TURN	TURN TO RIVER	TURN & RIVER
Pocket pair to set	4♦ 4♣	Q♦ 2♥ 9♠	4♥ 4♠	2	4.3% (22-1)	4.3% (22-1)	8.4% (11-1)
One overcard	A♠ 8♦	9♣ 5♠ 2♦	A♣ A♦ A♥	3	6.4% (14-1)	6.5% (14-1)	12.5% (7-1)
Inside straight draw	J♥ 10♣	Q♠ 8♦ 4♣	9♠ 9♦ 9♠ 9♥	4	8.5% (11-1)	8.7% (11-1)	16.5% (5-1)
Two pair to full house	J♥ 9♣	J♠ 9♦ 5♠	J♣ J♦ 9♥ 9♠	4	8.5% (11-1)	8.7% (11-1)	16.5% (5-1)
Two overcards to overpair	A♦ J♥	10♣ 8♦ 2♠	A♥ A♣ A♠ J♦ J♣ J♠	6	12.8% (13-2)	13$ (13-2)	24.1% (3-1)
Set to full house/ four of a kind	5♣ 5♦	5♠ 6♥ J♣	5♥ 6♦ 6♣ 6♠ J♣ J♦ J♥	7	14.9% (11-2)	15.2% (11-2)	27.8% (5-2)
Open-ended straight draw	9♣ 8♦	7♣ 6♥ 3♠	10♠ 10♦ 10♣ 10♥ 5♠ 5♦ 5♣ 5♥	8	17% (5-1)	17.4% (5-1)	31.5% (2-1)
Flush draw	Q♠ 10♠	A♠ 6♠ 8♦	2♠ 3♠ 4♠ 5♠ 7♠ 8♠ 9♠ J♠ K♠	9	19.1% (4-1)	19.6% (4-1)	35% (2-1)
Inside straight and flush draw	A♦ K♦	J♦ Q♠ 2♦	10♠ 10♦ 10♣ 10♥ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ Q♦	12	25.5% (3-1)	26.1% (3-1)	45% (5-4)
Open-ended straight and flush draw	K♥ Q♥	10♥ J♠ 3♥	9♠ 9♦ 9♣ 9♥ A♠ A♦ A♣ A♥ 2♥ 4♥ 5♥ 6♥ 7♥ 8♥ J♥	15	31.9% (2-1)	32.6% (2-1)	54.1% (5-6)

Figure 2: https://www.oddsshark.com/poker/strategy/math

pot odds and the odds of a drawing hand (derived from outs at showdown) gives an indication of a player's expected value of a call. The law of large numbers tells us a player will make money over a long period of time if they call with positive expected value. Using pot odds to inform one's choices can help players make more mathematically-informed decisions.

It is a common and useful tactic to manipulate pot odds by making a strong bet to protect a made hand that represents the nuts at the flop earlier. This is to discourage opponents from seeing the turn or river for a drawing hand by making the pot odds disfavourable to them. Note that this is not the same as bluffing as often one has a made hand.

For instance, suppose there was a \$20 pot in a heads-up. If player A bets \$10, player B odds will be 3:1 or 25% as it would cost \$10 to call. If player B has an open ended straight draw which has 17% chance of landing, it would make sense to fold. However, if the player bet \$2, then the odds are 9% to call, in which case it would make sense mathematically for player B to call.

Although it is important for players to maximize their expected earnings, having a fixed range for which one player either raises, calls, or folds can be easily exploited once other players pick up on opponents' patterns. For example, if a player is risk averse and only raises strong hands, while folding weak ones, the opponent can adopt a pure strategy of folding at every rise, while still betting their strong hands. However, the former player can pick up on this dynamic and start bluffing, knowing that the latter player folds easily. At the other end of the spectrum, a player who bluffs too many hands can easily be exploited by aggressive players who will make larger profits with a strong hand. Therefore, it is clear that a player should have a range of hands with which they bluff and should not bet proportionally to the strength of their hand.

4 Mixed Strategy Nash Equilibrium Poker

When players pick up on each other's strategies and start exploiting them, a Mixed Strategy Nash Equilibrium is reached where each player knows all other players' exact strategies, they all maximally exploit each other, and there is no useful deviating strategy to improve one's expected payoff. [Modern] Each player's purpose is to make the other player indifferent between bluffing and value-playing, or between checking and betting. Given the players' ranges, the MSNE can be computed, such that each player will have an optimal frequency of betting, checking, and folding, and an expected payoff for each hand.

5 Example MSNE

To see a MSNE in action, take for example two players with ranges AA, KK, QQ, meaning they will fold any hole cards outside of this list. Say, at the river, player 1 has KK, player 2 has AA or QQ each with 50% probability, while the community cards are not a factor determining the winner at showdown. Normalize the pot to 1$ and each player's remaining stack with 1$.

First, we eliminate the dominated actions:

1. Folding AA is dominated by betting AA. This is because AA always wins in this scenario, so its bet has positive expected value, while folding always gives 0 expected value.

2. Calling a bet with QQ is dominated by folding QQ. This is because QQ always loses so its expected value when betting is negative at showdown, while folding it gives 0 expected value.

3. Betting or raising KK is dominated by checking KK. If player 1 bets or raises with KK, player 2 either never folds if they have AA, in which case player 1 always loses, or they always fold QQ, in which case player 1 does not gain any value.

4. Checking AA is dominated by betting AA. If player 2 checks AA, player 1 never bets KK, so player 2 does not gain any value.

Thus, player 1 can only check or fold KK, while player 2 can only bet AA, or fold, check, or bet (bluff) QQ.

Pure strategy analysis:

1. If player 2 always bluffs QQ, player 1 can exploit that by always calling or checking, meaning they each win 50% of the time in the betting war. Thus EV[player 1] = 50%*(pot+player 2's bet)+50%*(-player 1's bet) = 0.5*2-0.5*1 = $ 0.5. Similarly EV[player 2] = $ 0.5.

2. If player 2 never bluffs QQ, player 1 can exploit that by never calling the bets, since a bet would mean player 2 has AA, so a certain loss. Thus player 2 will win 50% of the time only the value of the pot, and will otherwise fold, so will gain 0 value. EV[player 2] = 50%*pot = $ 0.5

We will see how player 2's payoff of \$ 0.5 is not optimal, by analyzing the MSNE. In equilibrium, player 2 will make player 1 indifferent to calling or folding KK. This means:

$$EV[P1] = EV[P1, call] = EV[P1, fold] = 0$$

But

$$\begin{aligned} EV[P1, call] &= \%win * (P1bet + pot) + \%lose * (-P1bet) \\ &= w(b+1) - (1-w)b \\ &= w(2b+1) - b = 0 \end{aligned} \tag{1}$$

So $(w = b/(2b+1)$. If the bets are all-in, $b = 1$, so $w == 33.3\%$.

The bluffing frequency of P2 must equal P1's pot odds, i.e. their probability of winning, such that P1 is indifferent between folding and calling. This means that P2 bluffs 33.3% of the time while playing value 66.7% of the time. So the bluff-to-value ratio is 1:2. Thus, the betting frequency of P2 is 75% since they bet 100% of the time when they get AA, and 50% of the time when they get QQ, so $50\% * 100\% + 50\% * 50\% = 75\%$.

$$EV[P2] = 50\% * EV[P2, AA] + 50\% * EV[P2, QQ]$$

Where:

$$\begin{aligned} EV[P2, AA] &= 1 * P(P1folds) + (1+1) * P(P1calls) \\ &= 1 * 0.5 + 2 * 0.5 \\ &= 1.5 \end{aligned} \tag{2}$$

$$\begin{aligned} EV[P2, QQ] &= 50\% EV[P2, QQ, bluff] + 50\% EV[P2, QQ, check] \\ &= 0.5 * (1 * P(P1folds) - 1 * P(P1calls)) + 0.5 * 0 \\ &= 0.5 * (1 * 0.5 - 1 * 0.5) \\ &= 0 \end{aligned} \tag{3}$$

Thus $EV[P2] = 0.5 * 1.5 + 0.5 * 0 = \$0.75 > \$0.5$. So the mixed strategy has higher gains for player 2.

Analyzing P1 actions, we conclude that whenever P2 checks it means that they have QQ since P2 only bets with AA, so P1 also checks and wins the round. Otherwise, we established earlier that P1 is indifferent between calling and folding, but in order to not be exploited, they also need a mixed strategy (for example if P1 always folds, P2 will always bluff). So P1 must make P2 indifferent between bluffing and checking. $EV[P2, check] = 0$, so $EV[P2, bluff] = 0$. Let calling frequency $= c$ and folding frequency $= 1 - c$. Then,

$$EV[P2, bluff] = pot * P(P1folds) + (-bet) * P(P1calls)$$

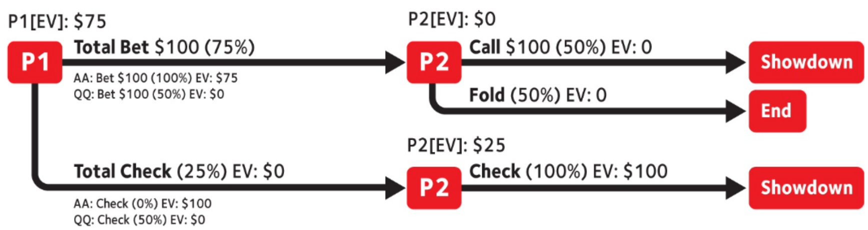


Figure 3: Modern poker theory: building an unbeatable strategy based on GTO principles, page 112

$$1 * (1 - c) - 1 * c = 0$$
$$c = 50\%$$

The calling frequency is also known as the minimum defense frequency.

$$\begin{aligned} EV[P1] &= EV[P1, call] * P(P1, call) + EV[P1, fold] * P(P1, fold) + EV[P1, check] * P(P1, check) \\ &= 0 + 0 + pot * 25\% \\ &= 0.25\$ \end{aligned} \tag{4}$$

6 Computing GTO Poker

To compute more complex MSNE, poker players can use software available such as:

1. Equity calculators. There are programs that can calculate the probability of winning with a hand, given some or all of the community cards, and given some or all of the opponents' cards. https://www.888poker.com/poker/poker-odds-calculator

2. GTO solvers. There are commercial solvers that can calculate Nash Equilibrium strategies for post-flop Limit and No-Limit Hold'em situations with arbitrary ranges and bet-sizes in any heads-up spot to a very high level of accuracy, meaning low exploitability.

7 References

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